Book of Abstracts

# Workshop
# Mathematical Analysis in Broad Perspective

Faculty of Mathematics, Informatics and Mechanics, University of Warsaw
Warsaw, 29–30 October 2018

Organizers: Benoît Perthame and Agnieszka Świerczewska-Gwiazda
Local Organizers: Tomasz Dębiec, Kamila Łyczek and Jakub Skrzeczkowski



# Schedule

| Monday, October 29<br>MIMUW, Room 2180 | Tuesday, October 30<br>MIMUW, Room 2180 |
|---|---|
| **9:00–9:10** Opening | **8:30–9:00** Coffee |
| **9:10–9:40**<br>Paweł Strzelecki<br>*Geometric curvature energies* | **9:00–9:30**<br>Mirosław Bulíček<br>*PDE analysis of a class of thermodynamically compatible viscoelastic compressible and incompressible rate-type fluids with stress-diffusion* |
| **9:45–10:15**<br>Benoît Perthame<br>*PDE models of natural networks* | **9:35–10:05**<br>Nastassia Pouradier Duteil<br>*Sparse control of Hegselmann-Krause models: Black hole and declustering* |
| **10:20–10:50** Coffee break | **10:10–10:40** Coffee break |
| **10:50–11:35**<br>Anna Marciniak-Czochra & Frits Veerman<br>*Mathematical modelling and analysis to understand mechanisms of tissue patterning* | **10:40–11:10**<br>Tomasz Dębiec<br>*Relative entropy method for measure solutions of the growth-fragmentation equation* |
| **11:40–12:10**<br>Fabrice Bethuel<br>*Concentration sets for multiple equal-depth wells potentials in the 2D elliptic case* | **11:15–11:45**<br>Didier Smets<br>*Linear stability of columnar vortices for 3D Euler* |
| **12:15–12:45**<br>Rafał Latała<br>*Norms of random matrices with independent entries* | **11:50–12:20**<br>Anna Zatorska-Goldstein<br>*Stationary heat flow on low dimensional rectifiable sets in $R^N$* |

| Monday, October 29<br>MIMUW, Room 2180 | Tuesday, October 30<br>MIMUW, Room 2180 |
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| **12:50–14:15** Lunch | **12:25–14:00** Lunch |
| | **14:00–14:30**<br>Piotr B. Mucha<br>*Around compressible flow models* |
| | **14:35–15:05**<br>Julien Guillod<br>*Nonlinear stability for the Ginzburg–Landau equation* |
| | **15:10–15:40** Coffee break |
| | **15:40-16:10**<br>Piotr Gwiazda<br>*A two species hyperbolic-parabolic model of tissue growth* |
| **18:30**<br>Conference Dinner | **16:15–16:45**<br>Agnieszka Świerczewska-Gwiazda<br>*General conservation laws: some new observations* |

## Concentration sets for multiple equal-depth wells potentials in the 2D elliptic case

**Fabrice Bethuel**

Laboratoire Jacques-Louis Lions, Sorbonne Université

### Abstract

The formation of codimension-one interfaces for multiwell gradient-driven problems is well-known and established in the scalar case, where the equation is often referred to as the Allen-Cahn equation. The vectorial case in contrast is quite open. This lack of results and insight is to a large extend related to the absence of known appropriate monotonicity formula. I will focus on the elliptic case in two dimensions, and describe some methods which allow to circumvent the lack of monotonicity formula.

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## PDE analysis of a class of thermodynamically compatible viscoelastic compressible and incompressible rate-type fluids with stress-diffusion

**Miroslav Bulíček**

Charles University

### Abstract

We present a system of PDEs governing the motion of non-Newtonian fluids described by a simplified viscoelastic rate-type model with a stress-diffusion term. The simplified model shares many qualitative features with more complex viscoelastic rate-type models that are frequently used in the modeling of fluids with complicated microstructure. As such, the simplified model provides important preliminary insight into the mathematical properties of these more complex and practically relevant models of non-Newtonian fluids. The simplified model that is analyzed from the mathematical perspective is shown to be thermodynamically consistent, and we extensively comment on the interplay between the thermodynamical background of the model and the mathematical analysis of the corresponding initial-boundary-value problem. Furthermore, we will show how the result can be extended to the non-simplified setting.

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## Relative entropy method for measure solutions of the growth-fragmentation equation

**Tomasz Dębiec**

University of Warsaw

**Abstract**

Entropy-based methods, and in particular the so-called "generalised relative entropy" inequalities, have been developed and successfully applied to structured population equations, and in particular to aggregation-fragmentation problems, over the last two decades.

In this talk, we study how entropy methods have been recently extended to measure solutions of the growth-fragmentation equation.

Joint work with Marie Doumic, Piotr Gwiazda and Emil Wiedemann.

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## Nonlinear stability for the Ginzburg–Landau equation

**Julien Guillod**

Sorbonne Université

**Abstract**

The real Ginzburg–Landau equation possesses a family of spatially periodic equilibria. If the wave number of an equilibrium is strictly below the so-called Eckhaus boundary the equilibrium is known to be stable. If the wave number is above the Eckhaus boundary the equilibrium is unstable. Exactly at the Eckhaus boundary spectral stability holds and the aim of this talk is to show that nonlinear stability still occurs. The limit profile is determined by a nonlinear equation since a nonlinear term turns out to be marginal w.r.t. the linearized dynamics. This is joint work with G. Schneider, P. Wittwer, and D. Zimmermann.

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**A two species hyperbolic-parabolic model of tissue growth**

**Piotr Gwiazda**

Institute of Mathematics, Polish Academy of Sciences, Warsaw

**Abstract**

Models of tissue growth are now well established, in particular in relation to their applications to cancer. They describe the dynamics of cells subject to motion resulting from a pressure gradient generated by the death and birth of cells, itself controlled primarily by pressure through contact inhibition. In the compressible regime we consider, when pressure results from the cell densities and when two different populations of cells are considered, a specific difficulty arises from the hyperbolic character of the equation for each cell density, and to the parabolic aspect of the equation for the total cell density. For that reason, few a priori estimates are available and discontinuities may occur. Therefore the existence of solutions is a difficult problem.

In a common work with Benoît Perthame and Agnieszka Świerczewska-Gwiazda [2] we established the existence of weak solutions to the model with two cell populations which react similarly to the pressure in terms of their motion but undergo different growth/death rates:

$$\begin{cases} \partial_t n_1 - \operatorname{div}[n_1 \nabla p] = n_1 F_1(p) + n_2 G_1(p), & x \in \mathbb{R}^d,\ t \geq 0, \\ \partial_t n_2 - \operatorname{div}[n_2 \nabla p] = n_1 F_2(p) + n_2 G_2(p), \end{cases}$$

with

$$n := n_1 + n_2, \qquad p = n^\gamma, \quad \gamma > 1.$$

We assume that there is a value $P_H > 0$ such that the smooth functions $F_i$, $G_i$, describing the division/death rates of cells, satisfy the properties

$$F(p) := F_1(p) + F_2(p) \leq 0, \qquad G(p) := G_1(p) + G_2(p) \leq 0, \qquad \forall p \geq P_H.$$

In opposition to the method used in the recent paper [1], our strategy is to ignore compactness on the cell densities and to prove strong compactness on the pressure gradient. We improve known results in two directions; we obtain new estimates, we treat higher dimension than one and we deal with singularities resulting from vacuum.

## References

[1] J. A. Carrillo, S. Fagioli, F. Santambrogio, and M. Schmidtchen. *Splitting schemes & segregation in reaction-(cross-)diffusion systems*, arxiv:1711.05434, (2017).

[2] P. Gwiazda, B. Perthame, A. Świerczewska-Gwiazda *A two species hyperbolic-parabolic model of tissue growth*, arXiv:1809.01867, (2018)

## Norms of random matrices with independent entries

**Rafał Latała**

University of Warsaw

**Abstract**

The spectral norm of any matrix is bigger than the largest Euclidean norm of its rows and columns. We show that for Gaussian matrices with independent entries this obvious bound may be reversed in average up to a universal constant. We will also discuss similar bounds for Schatten norms and other random matrices with independent entries. The talk is based on a joint work with Ramon van Handel (Princeton) and Pierre Youssef (Paris).

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## Mathematical modelling and analysis to understand mechanisms of tissue patterning

**Anna Marciniak-Czochra & Frits Veerman**

Heidelberg University

**Abstract**

Cells and tissue are objects of the physical world, and therefore they obey the laws of physics and chemistry, notwithstanding the molecular complexity of biological systems. What are the mathematical principles that are at play in generating such complex entities from simple laws? Understanding the role of mechanical and mechano-chemical interactions in cell processes, tissue development, regeneration and disease has become a rapidly expanding research field in the life sciences.To reveal the patterning potential of mechano-chemical interactions, we have developed two classes of mathematical models coupling dynamics of diffusing molecular signals with a model of tissue deformation. In this talk we present the two mathematical approaches to describe mechano-chemical patterning and compare the new models to the classical Turing patterns. We discuss analytical and numerical challenges of the proposed models and, in particular, propose a new approach based on geometric singular perturbation theory to investigate the emerging spatially heterogenous structures.

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## Around compressible flow models

**Piotr B. Mucha**

University of Warsaw

### Abstract

In my short talk I plan to discuss models related to the compressible flows. In particular the subject will be concentrated around the compressible Navier-Stokes system in the two space dimensional case. The idea of the analysis is related to considerations of large volume (bulk) viscosity. Then as it is expected the flow stays close to an incompressible motion. I would like to show new results in this area discussing also the possible limit cases.

The talk will be based on joint results with Raphael Danchin.

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## PDE models of natural networks

**Benoît Perthame**

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Laboratoire Jacques-Louis Lions, Sorbonne Université, CNRS, UPD, INRIA-Paris

### Abstract

Transportation networks have been widely produced by nature. Neural networks, leaf venatures, vascular system, roots are example which share the property that the network transports a fluid or a current. In some cases, the formation of the network, during development of the individual, is based on a prematrix which can be seen has a porous media. The pressure produced by a source acts on this prematrix and generates new vessels. Physicists have proposed PDE models for to describe this interaction under the form of a singular system of parabolic/elliptic type.

This lecture will present some mathematical features of this system: energy considerations and existence of solutions, multiple and singular steady states.

This work is a collaboration with J. Haskovec, P. Markowich and M. Schlottbom.

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## Sparse control of Hegselmann-Krause models: Black hole and declustering

**Nastassia Pouradier Duteil**

Laboratoire Jacques-Louis Lions, Sorbonne Université

**Abstract**

We elaborates control strategies to prevent clustering effects in opinion formation models. This is the exact opposite of numerous situations encountered in the literature where, on the contrary, one seeks controls promoting consensus. In order to promote declustering, instead of using the classical variance that does not capture well the phenomenon of dispersion, we introduce an entropy-type functional that is adapted to measuring pairwise distances between agents. We then focus on a Hegselmann-Krause type system and design declustering sparse controls both in finite-dimensional and kinetic models. We provide general conditions that characterize whether clustering can be avoided as function of the initial data. Such results include the description of black holes (where complete collapse to consensus is not avoidable), safety zones (where the control can keep the system far from clustering), basins of attraction (attractive zones around the clustering set) and collapse prevention (when convergence to the clustering set can be avoided).

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## Linear stability of columnar vortices for 3D Euler

**Didier Smets**

Laboratoire Jacques-Louis Lions, Sorbonne Université

**Abstract**

Columnar vortices are stationary solutions of the three-dimensional Euler equations with axial symmetry, where the velocity field only depends on the distance to the axis and has no component in the axial direction. Stability of such flows was first investigated by Lord Kelvin in 1880, but despite a long history the only analytical results available so far provide necessary conditions for instability under either planar or axisymmetric perturbations. We shall report on recent work with Thierry Gallay (Grenoble) in which we show that columnar vortices are spectrally stable with respect to three-dimensional perturbations with no particular symmetry.

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## Geometric curvature energies

**Paweł Strzelecki**

University of Warsaw

### Abstract

Various geometric curvature energies, including integral Menger curvature, and tangent point repulsive potentials, have striking connections with many branches of pure mathematics (harmonic analysis, variational calculus, geometric knot theory) and with applications for modelling of the physical objects without self-intersections (elastic rods, membranes, entangled or knotted DNA molecules, optimal packing issues etc.).

I shall present a brief survey of the properties of these energies, including their quantitative smoothing effects and their applications to variational and topological questions. I shall also discuss related open questions.

The talk will be based on joint research with Heiko von der Mosel and our former students.

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## General conservation laws: some new observations

**Agnieszka Świerczewska-Gwiazda**

University of Warsaw

### Abstract

A common feature of systems of conservation laws of continuum physics is that they are endowed with natural companion laws which are in such case most often related to the second law of thermodynamics. They are endowed with nontrivial companion conservation laws, which are immediately satisfied by classical solutions. Not surprisingly, weak solutions may fail to satisfy companion laws, which are then often relaxed from equality to inequality and overtake a role of a physical admissibility condition for weak solutions. We want to discuss what is a critical regularity of weak solutions to a general system of conservation laws to satisfy an associated companion law as an equality.

The second part of the talk will concern measure-valued solutions to systems of conservation laws. In the last years measure-valued solutions started to be considered as a relevant notion of solutions if they satisfy the so-called measure-valued – strong uniqueness principle. This means that they coincide with a strong solution emanating from the same initial data if this strong solution exists. Our goal is to provide a unified framework for general systems, that would cover the most interesting cases.

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# Stationary heat flow on low dimensional rectifiable sets in $R^N$

**Anna Zatorska-Goldstein**

University of Warsaw

## Abstract

I will discuss an elementary linear elliptic equation on a lower dimensional rectifiable structure in $R^N$ with Neumann boundary data. The set may be described by means of a finite Borel measure $\mu$ supported on it. This allows us to reformulate the equation and the boundary condition and to establish existence and uniqueness of a weak solution via a variational method. The setting requires an appropriate definition of a Sobolev-type space dependent on the measure $\mu$ and an appropriate Poincaré-type inequality. I will present examples of structures that are not manifolds and which do not support a global Poincaré inequality, yet which are admissible for our setting.